Final exam (duration: 2hr)
Financial maths and statistics

Check that this exam paper has 2 pages. This is a closed-book exam. No materials are permitted. Non-programmable calculators are allowed. All other computing, communicating, and electronic devices must be switched off and placed in your bag/backpack at the front of the examination room.

Exercises are independent from each other. Stick to the notation in the exercise if one is provided. Points will be awarded based on the solution method (formulas used, calculations) not merely on the final answer. Report all results rounded to two decimal places (4 for percentage rates) while intermediate computations should be carried out with at least 4 significant figures.

Exercise 1

Joan has borrowed 175 000 € as a mortgage loan on her house. The interest rate is at an annual percentage rate of 2.40 %, compounded monthly. The monthly payment required by the bank is 776.30 €.

1. What is the effective monthly rate and the effective annual rate?
2. How long would it take Joan to pay off her loan?

After 5 years, Joan found a bank willing to refinance her loan. The quoted annual percentage rate is 2.30 % with quarterly compouding for the same time to maturity.

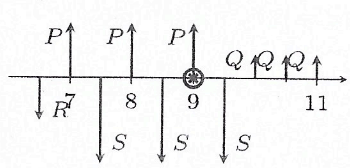
3. What is the outstanding balance after these first 5 years?
4. What would be the monthly payment on the new mortgage?
5. Should Joan still proceed if her former bank asks an early repayment charge of 3 % of the outstanding mortgage debt?

Exercise 2

1. A savings account is advertised to pay an annual percentage rate of 4 %, compounded semiannually.
   a. What is the effective annual rate?
   b. What is the effective interest rate over a month?
   c. What is the mortgage equivalent yield (MEY, annualized yield with monthly compounding)?
   d. What is the equivalent interest rate, compounded twice a month?

For each of the following questions, start with a cash flow diagram.

2. How much would you get after 10 years if you deposit 1000 € every quarter on this account?
3. How much should you pay every year starting at the end of the year for 3 years if you want to be able to withdraw 15 000 € in one year and a half and 7500 € in 4 years from now?
4. How long would it take to get 100 000 € on this account if you can start by depositing 2500 € at the end of the year and then keep depositing every 6 months from then on at a growth rate of 2 %?
5. Write the equation that relates the cash inflows to the cash outflows at the starred time in the following diagram:



Exercise 3

1. Give the expression of the expected return and variance of a portfolio made of assets A and B with weights w_A and w_B.

From now on, the two assets A and B are supposed to be perfectly negatively correlated.

2. Knowing that $w_B = 1 - w_A$, deduce from the previous question the expression of the volatility of the portfolio[1] when $w_A < \dfrac{\sigma_B}{\sigma_A + \sigma_B}$ or $w_A \geq \dfrac{\sigma_B}{\sigma_A + \sigma_B}$.

3. Obtain an equation relating the expected return of the portfolio $E(r_P) = \mu_P$ to its volatility σ_P in both cases.

The following data are available for assets A and B:

	Expected return (%)	Volatility (%)
A	7.00	3.50
B	10.50	4.45

4. Rewrite the previous relationship given these data. What should be the risk-free interest rate?

5. Give the weights w_A and w_B that would make the portfolio volatility equal to zero.

6. Make explicit the asset allocation for this risk-free portfolio if the invested capital is 500 000 €.

Formulas

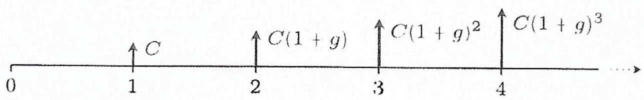
- The present value of a standard annuity of n cash flows, all with the same amount A, is given by

$$\text{PV} = \sum_{t=1}^{n} \frac{A}{(1+r)^t} = \frac{A}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

where r is the effective interest rate over one period.

- The present value of a growing annuity with n cash flows is

$$\text{PV} = \frac{C}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^n\right)$$



- If the IRR is between r_1 and r_2, a linear interpolation allows to get an approximate value:

$$\frac{r_1 - \text{IRR}}{r_1 - r_2} = \frac{\text{NPV}_1 - 0}{\text{NPV}_1 - \text{NPV}_2} \Leftrightarrow \text{IRR} = r_1 + \frac{(r_2 - r_1)}{(\text{NPV}_1 - \text{NPV}_2)} \times \text{NPV}_1$$

- The expected return of an asset indexed by i is given by $E(r_i) = \overline{r_i} = \mu_i$. The variance is denoted by $\text{Var}(r_i) = \sigma_i^2 = E(r_i^2) - \mu_i^2$.
- The covariance between returns r_i and r_j is defined as $\text{Cov}(r_i, r_j) = \sigma_{ij} = E(r_i r_j) - \mu_i \mu_j$ while their correlation is $\rho_{ij} = \dfrac{\sigma_{ij}}{\sigma_i \sigma_j}$.

[1]Use the fact that $a^2 + b^2 - 2\,ab = (a-b)^2$ and $\sqrt{x^2} = |x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$