Final exam (3h)
Financial maths and statistics

Exercises are independent from each other. Points will be awarded based on the solution method (formulas, computations), not merely on the final result. Report all results rounded to at least two decimal places while intermediate computations should be carried out with at least 4 significant figures. Any documentation/smartphone/computer is prohibited. Non-programmable calculator allowed.

Exercise 1

1. Starting from the present value of a standard annuity, give its future value (at the time of the last cash flow). How about growing annuities?

Twenty-five years before retiring, John wants to build up a supplementary pension provision. He finds a pension savings account earning an annual percentage rate of 6.3473 %, compounded quarterly. No withdrawals are allowed on this account before retirement.

2. What is the effective annual rate?

3. What is the equivalent interest rate, compounded monthly (r_m)? semiannually (r_s)?

Paul wants to be able to get an additional income of at least 6000 € twice a year for the first 12 years after he retires.

4. What is the value of his pension benefit upon retirement?

5. How much should his monthly contributions be to be able to meet his goals?

Often, contributions to pension savings accounts are a fraction f of income. At the moment of his first deposit, the salary of John is 2000 € . His salary will increase at a rate of 0.25 % every month until he retires.

6. What fixed percentage of his income (f) should he pay each month to meet his goals?

Exercise 2

You are a new intern and your boss asks you to review a previous analysis that was done to compare a few proposals to enhance the firm's manufacturing facility. You find that the prior analysis recommended the highest IRR option. Not only that, but a few data are missing (grayed-out cells). Here is the information you have (all figures are in thousands of $ unless noted otherwise):

Proposal	IRR (%)	Year 0	Year 1	Year 2	Year 3
A	38.20	-100	40	86.5	68
B	40.00	-100	40	0	159 + S
C	45.50	I	0	164	91
D	IRR_D	-125	86	86	86

1. Check that the IRR of proposal A is indeed 38.20 %.

2. For proposal B, what is the additional salvage value S that will be recovered at the end of year 3?

3. For proposal C, how much is the total initial investment I that was required in year 0?

4. What is the IRR of proposal D ?

5. Suppose the appropriate cost of capital for each alternative is 10 %. Determine the NPV of each project.

6. (Bonus) Which project should the firm choose? Why is taking the projects with the highest IRR not valid in this situation?

Exercise 3

An asset manager estimates that three scenarios are likely to occur at a horizon of one year: an economic expansion (E), a stable economy (S), and an economic downturn (D). She thinks that the three assets she considers investing in will behave as in the following tables:

Asset	Value today (£)	Value in one year (£)		
		E	S	D
F	50	60	55	44
G				15.75
H	20		22.5	

Probability of occurrence (%)	25	40	35
	E	S	D
Return on F (%)			
Return on G (%)	12.0	–8.0	5.0
Return on H (%)	5.0		–4.5

1. Many data are missing so reproduce these two tables and help her fill in the blanks.
2. Compute the expected return and the variance for each asset.
3. Compute the variance-covariance matrix of these asset returns.
4. What is the correlation between assets G and H?
5. Establish the general expression of the expected return and variance of a portfolio made only with two assets A and B with weights w_A and w_B in terms of the expected return, the volatility of each component return, and their correlation ρ_{AB}.
6. Deduce the weights of the minimum variance portfolio that can be made out of assets A and B (Hint: remember that $w_B = 1 - w_A$).
7. What are the characteristics of the minimum variance portfolio V given assets G and H (weights of each component asset, expected return and volatility)?
8. How should the asset manager allocate 5 000 000 £ in order to get the minimum variance portfolio?

Formulas

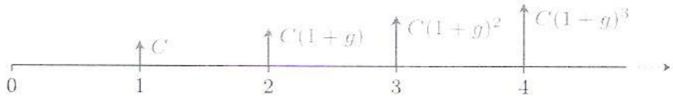
- The present value of a standard annuity of n cash flows, all with the same amount A, is given by

$$\text{PV} = \sum_{t=1}^{n} \frac{A}{(1+r)^t} = \frac{A}{r}\left(1 - \frac{1}{(1+r)^n}\right)$$

where r is the effective interest rate over one period.

- The present value of a growing annuity with n cash flows is

$$\text{PV} = \frac{C}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^n\right)$$



- If the IRR is between r_1 and r_2, a linear interpolation allows to get an approximate value:

$$\frac{r_1 - \text{IRR}}{r_1 - r_2} = \frac{\text{NPV}_1 - 0}{\text{NPV}_1 - \text{NPV}_2} \Leftrightarrow \text{IRR} = r_1 + \frac{(r_2 - r_1)}{(\text{NPV}_1 - \text{NPV}_2)} \times \text{NPV}_1$$

- The expected return of an asset indexed by i is given by $E(r_i) = \overline{r_i} = \mu_i$. The variance is denoted by $\text{Var}(r_i) = \sigma_i^2 = E(r_i^2) - \mu_i^2$.
- The covariance between returns r_i and r_j is defined as $\text{Cov}(r_i, r_j) = \sigma_{ij} = E(r_i r_j) - \mu_i \mu_j$ while their correlation is $\rho_{ij} = \dfrac{\sigma_{ij}}{\sigma_i \sigma_j}$.